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Verified NTT Multiplications for NISTPQC KEM Lattice Finalists: Kyber, SABER, NTRU

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Postquantum Cryptography (PQC)

- A large-scale quantum computer breaks RSA and ECC by Shor's algorithm
- New cryptosystems that withstand quantum computing are required
    - Postquantum Cryptography (PQC)
- PQC standardization process (NISTPQC) initiated by NIST
    - 7 finalists (KYBER, SABER, NTRU, ...) and 8 alternate candidates in the 3rd round

Implementation Issues

- Cryptography is always under a lot of pressure to be efficient
- Every round-3 submission in NISTPQC includes hand-optimized software
- PQC tends to be also more complex than pre-quantum public-key cryptography
- Bugs in PQC implementations?

Formal Verification

- Consider the field multiplication over $\mathbb{F}_p$ with $p = 2^{255} - 19$.
- There are roughly $2^{510} (= 2^{255} \times 2^{255})$ different inputs.
- How many of them can be tested?
  - What about those inputs which are never tested?
  - "Testing shows the presence, not the absence of bugs."

E. W. Dijkstra (1969)

- Formal verification aims to prove the absence of bugs through logical or mathematical reasoning.
  - That is, the field multiplication is computed correctly for *all* inputs.

Functional Correctness

- Testing only checks that an implementation is correct on a fixed set of selected inputs
- Formal verification can reach a conclusion that the implementation computes the correct outputs for all possible inputs
- CryptoLine[1] was developed to help programmers write correct cryptographic assembly programs
  - A domain-specific language for modeling cryptographic assembly programs and their specifications
  - A tool for verifying programs in the domain-specific language
  - Support two kinds of predicates
    - Algebraic predicates: non-linear (modular) equations over integers
    - Range predicates: bit-accurate comparisons, equations, or modular equations

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[1] `https://github.com/fmlab-iis/cryptoline`

Our Contributions

- First verification of highly complex polynomial multiplications based on the Number Theoretic Transform (NTT)

	Intel AVX2		ARM Cortex-M4	
NTRU	ntt-polymul[2]	build 3e42ffa	pqm4[3]	build d26fee0
Kyber	PQClean[4]	build 688ff2f	pqm4[3]	build 688ff2f
SABER	ntt-polymul[2]	build 3e42ffa	Strategy A by [ACC+22][5]	

- Extension of the CryptoLine tool
  - Verification either much slower or impossible without these extensions

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[2] https://github.com/ntt-polymul/ntt-polymul
[3] https://github.com/mupq/pqm4
[4] https://github.com/PQClean/PQClean
[5] https://github.com/multi-moduli-ntt-saber/multi-moduli-ntt-saber

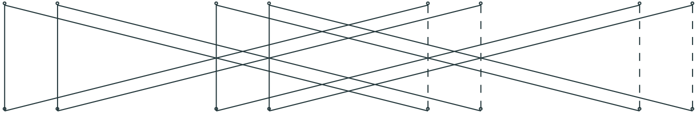
AVX2 KYBER768 NTT

- The incomplete NTT in the Intel AVX2 implementation from PQClean does the following map:

$$\begin{array}{lll} & \mathbb{Z}_q[X]/\langle X^{256}+1\rangle & \\ \to & \mathbb{Z}_q[X]/\langle X^{128}-\omega_4\rangle \times \mathbb{Z}_q[X]/\langle X^{128}+\omega_4\rangle & \text{(level 0)} \\ \to & \cdots & \vdots \\ \to & \mathbb{Z}_q[X]/\langle X^{2}-\zeta_{6,0}\rangle \times \cdots \times \mathbb{Z}_q[X]/\langle X^{2}-\zeta_{6,127}\rangle & \text{(level 6)} \end{array}$$

  where $\zeta_{i,j}$ is the roots of unity used at the end of level i (counting up)
- Cut at each level to decompose the verification problem

Workflow of Verifying AVX2 KYBER768 NTT



$F \equiv G_{0,0} \mod [q, X^{128} - \omega_4]$ $\qquad$ $F \equiv G_{0,1} \mod [q, X^{128} + \omega_4]$

$\vdots$ $\qquad$ $\vdots$

$F \equiv G_{i,j} \mod [q, X^{\frac{256}{2^{i+1}}} - \zeta_{i,j}], 0 \leq j < 2^i$ $\qquad$ $F \equiv G_{i,j} \mod [q, X^{\frac{256}{2^{i+1}}} - \zeta_{i,j}], 2^i \leq j < 2^{i+1}$

$\vdots$ $\qquad$ $\vdots$

$F \equiv G_{6,j} \mod [q, X^2 - \zeta_{6,j}], 0 \leq j < 64$ $\qquad$ $F \equiv G_{6,j} \mod [q, X^2 - \zeta_{6,j}], 64 \leq j < 128$

- All 256 coefficients used in level 0; at most 128 needed at level 1 onwards

Verification of AVX2 KYBER768 NTT i

Step 1: running trace (in assembly)
Extract from an executable by our script `itrace.py`

```
$ itrace.py test ntt PQCLEAN_KYBER768_AVX2_polyvec_ntt.gas
$ more  PQCLEAN_KYBER768_AVX2_polyvec_ntt.gas
#PQCLEAN_KYBER768_AVX2_polyvec_ntt:
:
# [some bookkeeping information]
:
        vmovdqa (%rsi),%ymm0            #! EA = L0x5555556395e0; Value = ...
        vpbroadcastq 0x140(%rsi),%ymm15 #! EA = L0x555555639720; Value = ...
        vmovdqa 0x100(%rdi),%ymm8       #! EA = L0x7fffffffb080; Value = ...
:
        vpbroadcastq 0x148(%rsi),%ymm2  #! EA = L0x555555639728; Value = ...
        vpmullw %ymm15,%ymm8,%ymm12     #! PC = 0x55555556eb85
:
        vpmulhw %ymm2,%ymm8,%ymm8       #! PC = 0x55555556eb99
:
```

Verification of AVX2 KYBER768 NTT ii

Step 2: Define translation between assembly and CRYPTOLINE instructions
Translation rules (usually standard and reusable)

```
#! $1c(%rsi) = %%EA
#! (%rsi) = %%EA
#! $1c(%rdi) = %%EA
#! (%rdi) = %%EA
#! %ymm$1c = %%ymm$1c
#! vpbroadcastq $1ea, $2v -> mov $2v_0 $1ea;\nmov $2v_1 $1ea[+2];\n
   mov $2v_2 $1ea[+4];\nmov $2v_3 $1ea[+6];\nmov $2v_4 $1ea;\n
   mov $2v_5 $1ea[+2];\nmov $2v_6 $1ea[+4];\nmov $2v_7 $1ea[+6]; ...
#! vmovdqa $1ea, $2v -> mov $2v_0 $1ea;\nmov $2v_1 $1ea[+2];\n
   mov $2v_2 $1ea[+4];\nmov $2v_3 $1ea[+6]; ...
#! vmovdqa $1v, $2ea -> mov $2ea $1v_0;\nmov $2ea[+2] $1v_1;\n
   mov $2ea[+4] $1v_2;\nmov $2ea[+6] $1v_3; ...
```

Verification of AVX2 Kyber768 NTT iii

Step 3: `to_zdsl.py` translates running trace to CryptoLine program

```
proc main( [inputs] ) =
{ [precondition to be defined] }
(* vmovdqa (%rsi),%ymm0            #! EA = L0x5555556395e0; ... *)
mov ymm0_0 L0x5555556395e0;
:
mov ymm0_f L0x5555556395fe;
(* vpbroadcastq 0x140(%rsi),%ymm15 #! EA = L0x555555639720; ... *)
mov ymm15_0 L0x555555639720;
:
mov ymm15_f L0x555555639726;
(* vmovdqa 0x100(%rdi),%ymm8       #! EA = L0x7fffffffb080; ... *)
:
{ [postcondition to be defined] }
```

Verification of AVX2 KYBER768 NTT iv

Step 4: Initialize constants used in the subroutine

```
(*********** constants ***********)
mov L0x5555556395e0 (  3329)@sint16; mov L0x5555556395e2 (  3329)@sint16;
...
mov L0x555555639600 ( -3327)@sint16; mov L0x555555639602 ( -3327)@sint16;
...
mov L0x555555639620 ( 20159)@sint16; mov L0x555555639622 ( 20159)@sint16;
...
mov L0x555555639adc (    32)@sint16; mov L0x555555639ade (    32)@sint16;
...
```

Verification of AVX2 KYBER768 NTT V

Step 5: pre-condition, the post-condition, and mid-conditions
(mid-conditions not required for AVX2 KYBER768 NTT, easy to generate using a script, result in less verification time)

Precondition
$-q < f_i < q$ for all $0 \leq i < 256$ where f_i's are the inputs and $q = 3329$

Midconditions and postcondition

$$F \equiv G_{i,j} \quad \bmod [q, X^{256/2^{i+1}} - \zeta_{i,j}] \text{ for all } 0 \leq j < 2^{i+1}$$

and

$$-(2+i)q < g_{i,j,k} < (2+i)q \text{ for all } 0 \leq j < 2^{i+1}, 0 \leq k < 256/2^{i+1}.$$

where i is the NTT level (from 0 to 6)

Verification of AVX2 KYBER768 NTT vi

Step 6: Run CRYPTOLINE, wait (human interaction no longer needed)

```
$ cv -v -isafety -jobs 24 -slicing -no_carry_constraint \
    PQCLEAN_KYBER768_AVX2_polyvec_ntt.cl
Parsing Cryptoline file:                [OK]            0.089273 seconds
Checking well-formedness:               [OK]            0.031599 seconds
Transforming to SSA form:               [OK]            0.019121 seconds
Rewriting assignments:                  [OK]            0.020577 seconds
Verifying program safety:               [OK]            183.994889 seconds
Verifying range assertions:             [OK]            42.385435 seconds
Verifying range specification:          [OK]            200.594131 seconds
Rewriting value-preserved casting:      [OK]            0.001421 seconds
Verifying algebraic assertions:         [OK]            0.007455 seconds
Verifying algebraic specification:      [OK]            26.648724 seconds
Verification result:                    [OK]            453.802915 seconds
```

Classical Compositional Reasoning

- Consider the following program snippet:

$$\begin{aligned} \text{cut}: \quad & P_0 \wedge P_1 \wedge \cdots \wedge P_{127} \\ & \text{[code]} \\ \text{cut}: \quad & Q_0 \wedge Q_1 \wedge \cdots \wedge Q_{127} \\ & \text{[code]} \\ & \ldots \end{aligned}$$

- It happens in inverse NTT that Q_i only depends on P_i but Q_i, P_i, and many other P_j's involve common variables
  - Those P_j's cannot be excluded systematically when verifying Q_i
  - Verification is quite inefficient or even impossible in such cases
- Proposed solution: nonlocal compositional reasoning

In Nonlocal Compositional Reasoning

- Each cut instruction is assigned to a number for reference
- Verifiers can add relevant premises by cut numbers

cut 0 : $P_0 \wedge P_1 \wedge \cdots \wedge P_{127}$

cut 1 : P_0 prove with 0

cut 2 : P_1 prove with 0

…

cut 128 : P_{127} prove with 0

cut 129 : true

[code]

…

cut 130 : Q_0 prove with 1 $\wedge$ Q_1 prove with 2 $\wedge \cdots$

[code]

…

Twisted NTT

- Mapping $X = aY$ from $\mathbb{F}[X]/\langle X^n - c\rangle$ to $\mathbb{F}[Y]/\langle Y^n - 1\rangle$ is called *twisting*

$$\frac{\mathbb{F}[X]}{\langle X^{2n} - 1\rangle} \cong \frac{\mathbb{F}[X]}{\langle X^{n} - 1\rangle} \times \frac{\mathbb{F}[X]}{\langle X^{n} + 1\rangle} \overset{X=aY}{\cong} \frac{\mathbb{F}[X]}{\langle X^{n} - 1\rangle} \times \frac{\mathbb{F}[Y]}{\langle Y^{n} - 1\rangle}$$

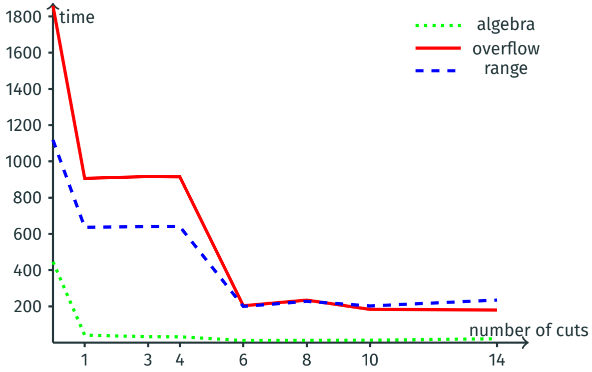
- Two approaches of specifying twisted NTT
  - With fresh variables $Y_{i,j}$ (ARM Cortex-M4 SABER)
  - Without fresh variables (Intel AVX2 SABER)

Verification Results (in Seconds)

KEM	*architecture*	*direction*	*algebra*	*overflow*	*range*	*total*
Kyber768	AVX2	normal	26.6	183.9	242.8	453.8
		inverse	761.7	781.0	6050.0	7593.5
	Cortex M4	normal	134.3	173.7	191.0	499.4
		inverse	1481.0	348.6	184.1	2014.3
`ntru2048509`	AVX2	normal	478.4	1229.8	1738.6	3447.8
		inverse	3868.6	1545.3	12170.3	17585.7
	Cortex M4	normal	1353.0	5970.7	4810.2	12135.2
		inverse	11315.1	3019.6	7813.7	22150.9
Saber	AVX2	normal	60.1	207.7	271.7	539.9
		inverse	436.2	443.8	859.4	1741.0
	Cortex M4	normal	110.2	2731.9	2196.7	5039.3
		inverse	3250.5	2754.0	853.4	6858.8

min: 453.8 seconds (≈ 8 minutes)
max: 22150.9 seconds (≈ 6 hours)

Effectiveness of Cuts in Intel AVX2 KYBER768 NTT



Human Time

- Each of our verifications took less than a week of calendar time
- The majority of it was really communication with the programmer of the code, and secondly reading and gaining a basic understanding of the program at hand

Conclusion

- We demonstrate the feasibility for a programmer to verify his or her high-speed assembly code for PQC
- We demonstrate the feasibility for a verification specialist to verify someone else's high-speed PQC software in assembly code, with some cooperation from the programmer
- Enhanced compositional reasoning techniques take full advantage of clearly demarcated stages in many cryptographic algorithms
- We did find a few bugs in high-speed software

Future Work

- The same technique applies to also
  - any implementation of small ideal-lattice-based cryptosystems that also has NTT-based arithmetic, e.g., the KEMs NTRU Prime, LAC, or NewHope and the signatures Dilithium and Falcon
  - a myriad of other architectures and other parameter sets
- Extend CRYPTOLINE to other PQCs such as Rainbow/UOV and Classic McEliece
- Watch out, we can do symmetric cryptography soon!

Thank you for listening